The application of unilateral single value control chart based on lognormal distribution

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Abstract

The risk theory tells us that the short-term high risks exist at the right tail of the distribution. So we advocate the design and parameter estimation method, which the special figures that reflect short-term risks are within the control limits of unilateral single value control chart based on lognormal distribution, and we also put forward how to use unilateral single value control chart through examples. This essay aims to monitor the short-term risks by means of control chart and reduce unnecessary losses of high risks.

Keywords: risk control, lognormal distribution, unilateral single value control chart

1 Introduction

Short-term risk assessment is one of the important parts of risk theory [1]. Present work mostly establish short-term risk models to describe the fluctuation process of the risk, while the research on the short-term risk fluctuation monitor is rarely seen. The control chart method in the quality management is an important method to monitor the short-term risk. The normal distribution that special figures reflect the short-term risks mostly shows right-side character (for example, lognormal distribution). High risk is right-skewed. Recent researches prove that the risk fluctuation monitor of the right-skewed distribution has its theoretical and practical significance.

Control chart is one of the seven quality engineer tools. Since Shewhart put forward normal control chart in 1924 to 1980s, the research on the method of control chart has made some progress.Shewhart control chart takes an important part in improving quality and productivity since included in the international standard. In the international standards ISO8258: 1991 [2] and GB / T4091-2001 [3]. Metering control chart what is assumed to quality characteristics follows a normal distribution. As for the problem of monitoring during the process of mass production, control chart contributes to improving the quality of products. However, putting SPC (Statistics Process Control) into use for short-term risk assessment has great limitations. Because mostly the random variable reflecting the short-term risk obeys right-skewed distribution.While the normal distribution $N(\mu,\sigma^2)$ is symmetry. And the high risks concentrate in the right of the distribution, so making control chart to monitor risks just needs making UCL (Upper Control Limit). This essay is intended to research unilateral single value control chart based on lognormal distribution and give analysis of example.

Control chart is graphical verification of assumptions. There are two kinds of errors in the verification of assumptions: one is α (abandon the right) and the other is β (get the wrong). In the theory of control chart, α is accidental and unavoidable while β is non-accidental and avoidable. We can detect the abnormal factors, promptly warn and reduce loses by means of the control chart.

In short-term risks control, α is the probability that actual risk loses should be claimed but not, while β is the probability that actual risk loses does not happen but claimed. These two errors will both cause losses to the insurance company, but the change direction of α is opposite to β. As a result, during making the control chart we should submit the principle that α plus β is the least. In this chapter we still use 3σ rules, α=0.0027.

2 Unilateral single value control chart

Suppose the special figure reflecting the short-term risk, X obeys lognormal distribution $LogN(\mu,\sigma^2)$. According to probability theory [4], the density function of lognormal distribution is

$$f(x;\mu,\sigma^2)=\begin{cases}\dfrac{1}{\sqrt{2\pi}\sigma}\dfrac{1}{x}e^{-\frac{1}{2}(\frac{\ln x-\mu}{\sigma})^2}, & x\ge 0\\ 0, & x<0\end{cases} \tag{1}$$

$f(\bullet)$ is the density function of lognormal distribution, μ σ^2 are parameters and $\ln(X)\sim N(\mu,\sigma^2)$. Given α, X_α is upper α quantile in $LogN(\mu,\sigma^2)$, and $P(X\le X_\alpha)=1-\alpha$. According to Shewhart control chart, bilateral single value control chart $Y\sim N(\mu,\sigma^2)$

$$\text{UCL}=\mu+Z_{\alpha/2}\sigma \qquad \text{CL}=\mu \qquad \text{LCL}=\mu-Z_{\alpha/2}\sigma, \tag{2}$$

UCL is the upper control limit, CL is control center line, LCL is Lower control limit, Z_α is upper α quantile in $N(0,1)$. From (2) we can see UCL=$\mu+Z_\alpha\sigma$ actually is Y's $\alpha/2$ quantile, LCL=$\mu-Z_{\alpha/2}\sigma$ is Y's 1-$\alpha/2$ quantile, CL=μ is Y's mathematical expectation $E(Y)$. So the control limit of $X\sim LogN(\mu,\sigma^2)$'s unilateral single value control chart is

$$\text{UCL}=X_\alpha \qquad \text{CL}=E(X) \tag{3}$$

$E(X)=e^{\mu+\sigma^2/2}$, just find the expression of X_α. By $P(X \le X_\alpha)=1-\alpha$, we get $P(\ln(X)\le \ln(X_\alpha))=1-\alpha$, and $\ln(X)\sim N(\mu,\sigma^2)$, $\ln(X_\alpha)$ is the quantile of normal distribution $N(\mu,\sigma^2)$. Coordinate transformation $Z_\alpha=(\ln(X_\alpha)-\mu)/\sigma$, given α=0.0027, from Standard normal distribution table $Z_{0.0027}=2.78$, so $X_\alpha=e^{2.78\sigma+\mu}$ we get

$$\text{UCL}=e^{2.78\sigma+\mu} \qquad \text{CL}=e^{\mu+\sigma^2/2} \tag{4}$$

When μ, σ is known, (4) is $X \sim LogN(\mu,\sigma^2)$ unilateral single value control chart (α=0.0027). When μ, σ is unknown,it comes below.

Suppose there are many loss samples $X_1,X_2,\cdots,X_n$, and $X_i \sim LogN(\mu,\sigma^2)$, $i=1,2,\cdots,n$. Then take the log of $X_1,X_2,\cdots,X_n$, we get $\ln(X_1),\ln(X_2),\cdots,\ln(X_n)$. $\ln(X_i)\sim N(\mu,\sigma^2)$. When

$$\overline{\ln(X)}=\frac{1}{n}\sum_{i=1}^{n}\ln(X_i),\ \rho_i=X_{(i)}/X_{(i+1)},$$

$$\ln(\rho_j)=|\ln(X_i)-\ln(X_{i+1})|,\ \overline{\ln(\rho)}=\frac{1}{n-1}\sum_{j=1}^{n-1}\ln(\rho_j),$$

$$j=1,2,\cdots,n-1.$$

According to Shewhart's one-value range control chart parameters estimation method [5], we get

$$\hat{\mu}=\overline{\ln(X)} \qquad \hat{\sigma}=\frac{\sqrt{\pi}}{2}\overline{\ln(\rho)}. \tag{5}$$

We can prove $E(\hat{\mu})=\mu$, $E(\hat{\sigma})=\sigma$, $\overline{\ln(X)}$ is unbiased estimation of μ. $\frac{\sqrt{\pi}}{2}\overline{\ln(\rho)}$ is unbiased estimation of σ. Put (5) into (4), we get the lognormal distribution's unilateral single value control chart when μ, σ is unknown.

3 Application

During an insurance company inspection period, there are 30 policies claimed. The short-term risk individual claim is table 1, and through former experience, individual claim obeys lognormal distribution. For example unilateral single value control chart application of step is below

TABLE 1 Individual claim. Unit: Ten thousand yuan

Num	Claim amount	Num	Claim amount	Num	Claim amount
1	30.55	11	30.36	21	14.97
2	8.94	12	11.70	22	13.75
3	10.35	13	17.94	23	5.96
4	4.42	14	25.06	24	21.21
5	10.27	15	21.23	25	4.46
6	5.42	16	1.97	26	9.51
7	16.42	17	40.93	27	14.71
8	19.59	18	24.08	28	15.98
9	1.71	19	4.03	29	7.01
10	4.95	20	3.49	30	17.46

Step 1: Calculate $\overline{\ln(X)}=2.09$, $\overline{\ln(\rho)}=0.86$. Estimate parameters $\hat{\mu}=2.09$, $\hat{\sigma}=0.76$;

Step 2: Calculate control limit from (4), $\text{UCL}=66.87$, $\text{CL}=10.79$;

Step 3: Draw control line, trace points, then we get control chart of analysis, which is shown in Fig. 3.1;

Step 4: Judge whether the process of monitoring is steady through the analysis control chart drawn from step3. All sample points in Fig.3.1 are within the control limit and exist no non-random phenomenon,so we suggest that the process is under control;

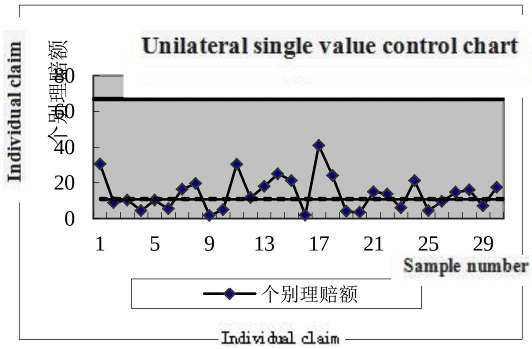

FIGURE 1 Unilateral single value control chart

Step 5: Prolong the control limit of control chart in step 3 and finally we get the control chart of control.

4 Conclusions

Control chart is an efficient tool to control risks. With the quality characteristic value in the lognormal distribution, the unilateral single value control chart given in the context, which reflects the short-term risks, is designed for the right distribution that high short-term risk exists. By monitoring the short-term risks, if charts appear the exception during the monitoring, it means the policy may have the risks of abnormal settlement of claims. Once this happens, you should find the causes (changes of the assurers' risks features or ethical risks etc.) ahead of time and give adjustments to cut down the losses in the future.

Comparing with other risk management devices, control chart is easy to operate. The operator can begin to work after simple training. Also, it can prevent the risk because the monitoring is for the process of the fluctuation of the risk.

References

[1] Xie Zhigang, Han Tianxiong 2000 *The Theory Of Risk And Non-Life Actuarial Mathematics* Nankai University Press **2000**(9) 75-80
[2] ISO 8258. Shewhart Control Charts 1991
[3] GB/T 4091-2001 *Shewhart Control Charts* Beijing:Standards Press of China 2001
[4] Sheng Ju, Xie Shiqian, Pan Chengyi 2001 *Probability Theory And Mathematical Statistics* Beijing:Higher Education Press 83-90
[5] Sun Gongxu, Sun Jing 2003 *New Edition Of Quality Management (Second Edition)* Beijing: Higher Education Press 75-128

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